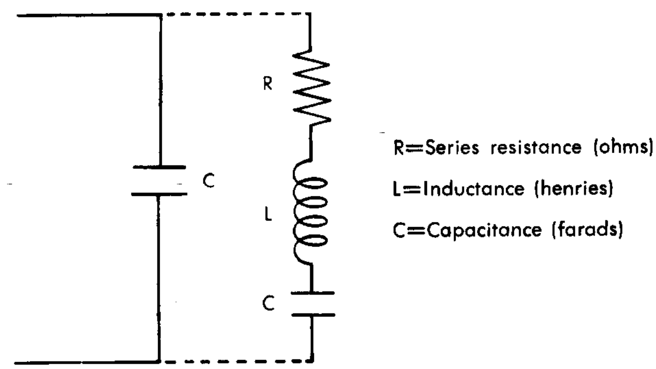
CAPACITORS…PF, DF & Q

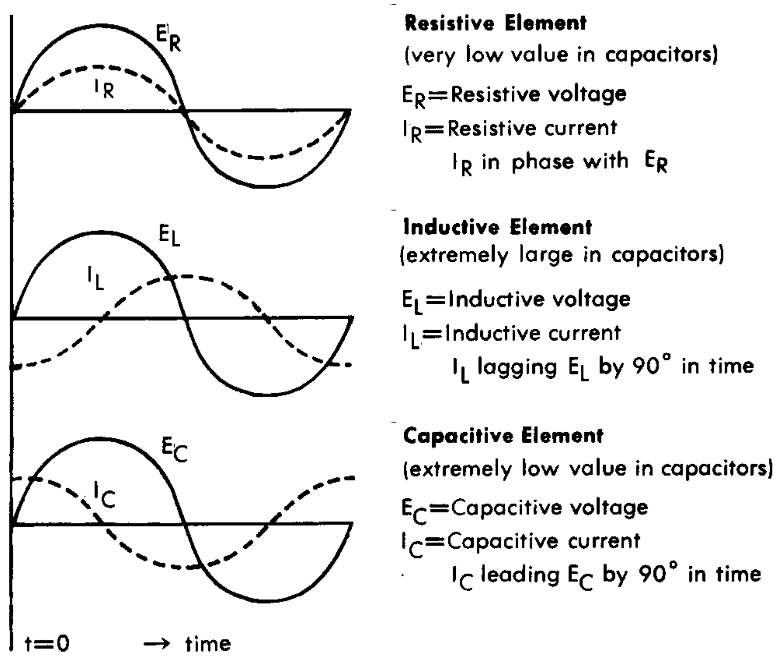
In any electrical device, including capacitors, when a certain amount of total power (energy) is applied to the device, a lesser amount passes out of it. The difference between the amounts in and out is "lost" or used within the device, and is referred to as the "power loss." If the "power loss" is divided by the input power, the resulting ratio is the "power factor" of the device. The power factor (PF) is a direct measurement of the **inefficiency** of a capacitor, which is a measuring tool to determine the wasted consumption of power by the capacitor itself, since such power is unavailable to do otherwise "useful" work.

A review of some fundamental concepts relative to the application of a sine wave voltage to a capacitor will be helpful.

Although a capacitor is primarily "capacitive" in nature, it does possess very small distributed amounts of resistive and inductive elements. These distributed amounts of resistive and inductive elements can be lumped into a single value for computation purposes as shown in the following equivalent circuit.



The result is a circuit containing all three primary elements of resistance, inductance, and capacitance. This means that the application of a sine wave voltage to the capacitor will set up a fundamental vector relationship between the voltages and currents in each element as follows:



Note also, that since R, L, & C are in series in the circuit, current (I) is the same current that passes through all elements (in magnitude) and therefore:

$$I = I_R = I_L = I_C \text{ (Magnitude)}$$

With the element currents out of phase with the voltages, Ohm's law must be modified to use impedances and reactances in addition to resistance. That is:

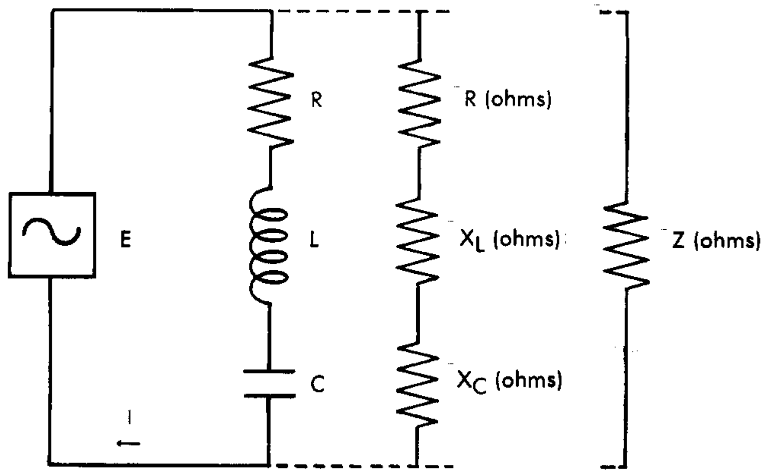
$E_R = IR$

$E_L = IX_L$ where $X_L = 2\pi fL$ (Inductive reactance in ohms)

$E_C = IX_C$ where $X_C = \frac{1}{2\pi fC}$ (Capacitive reactance in ohms)

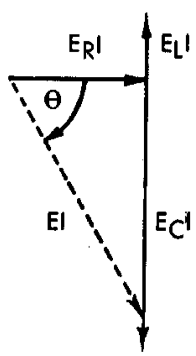
$E = IZ$ where Z=Total impedance of R, X_L, X_C combination in ohms

The power relationship of our circuit is:



EI = total circuit power (Θ)
$E_R I$ = Resistive power (0°)
$E_L I$ = Inductive power (—90°)
$E_C I$ = Capacitive power (+90°)

Drawing the vector relationship, the power equation is:



$$(EI)^2=(E_R I)^2+(E_C I-E_L I)^2$$

Substituting:

$$(IZI)^2=(IRI)^2+(IX_C I-IX_L I)^2$$

$$(I^2Z)^2=(I^2R)^2+(I^2X_C-I^2X_L)^2$$
$$=I^4R^2+I^4(X_C-X_L)^2$$

$$I^4Z^2=I^4[R^2+(X_C-X_L)^2]$$

Therefore: $Z^2=R^2+(X_C-X_L)^2$ and $Z=\sqrt{R^2+(X_C-X_L)^2}$

POWER FACTOR

The "power loss" within the capacitor is that portion of the total power applied to the capacitor that is not stored by the capacitor on an instantaneous basis. As the current passes through the series resistance element, it generates heat, which is the "power loss." It should be noted that all energy losses (due to leads, dielectric polarization, connections, and eddy currents in the electrode material) are taken into account by the "equivalent series resistance" element (see Technical Bulletin #10). Analysis of the power equation shows:

$$(EI)^2=(E_R I)^2+(E_C I-E_L I)^2$$

substituting: $(I^2Z)^2 =(I^2R)^2+(I^2X_C-I^2X_L)^2$

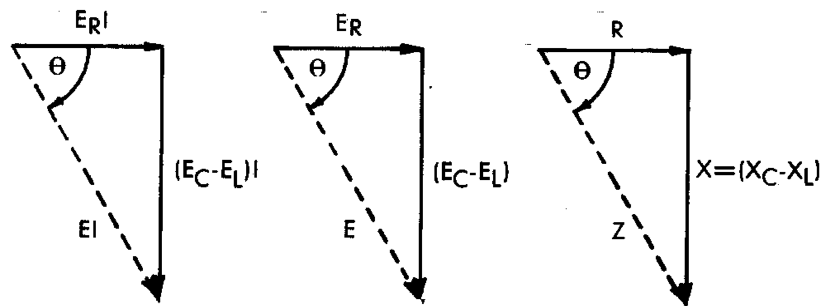
$$(\text{Power in})^2 = (\text{Power losses})^2 + (\text{power out})^2$$

And therefore the power factor by definition is:

$$PF=\frac{\text{Power Losses}}{\text{Power In}}=\frac{I^2R}{I^2Z}=\frac{E_R I}{EI}=\cos\theta$$

(from the vector diagram)

Note also that the following vector relationships hold true:



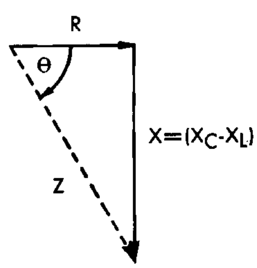
Power Diagram Voltage Diagram Impedance Diagram

$$PF=\cos\theta=\frac{E_R I}{EI}=\frac{E_R}{E}=\frac{R}{Z}$$

As shown, the simplest form for power factor is the ratio of the equivalent series resistance to the impedance of the device.

DISSIPATION FACTOR

With the exception of electrolytic and large power capacitors, most wound dielectric type units utilize a ratio figure known as the "Dissipation Factor" (DF) instead of the "Power Factor" (PF). By definition, this DF is the ratio of the equivalent series resistance to the reactance. For comparison purposes it can be noted that:



$$PF=\frac{R}{Z}=\cos\theta$$

$$DF=\frac{R}{X}=\cot\theta$$

This illustration shows that if X and Z are practically identical (which would be the case when ø approaches 90°), then the PF and DF would also be practically identical.

For any given unit, analysis of the divergence error in the equation DF=PF shows:

$$DF = \frac{R}{X} \qquad PF = \frac{R}{Z}$$

$$\therefore R = (DF)X \qquad \therefore R = (PF)Z$$

And: $(DF)X = (PF)Z = (PF)\left(\sqrt{R^2 + X^2}\right) = (PF)\left(\sqrt{(DF)^2X^2 + X^2}\right)$

$$(DF)X = (PF)\left(X\sqrt{(DF)^2 + 1}\right)$$

$$(DF) = (PF)\left(\sqrt{(DF)^2 + 1}\right)$$

Squaring: $(DF)^2 = (PF)^2 \left[(DF)^2 + 1\right] = (PF)^2 (DF)^2 + (PF)^2$

Rearranging: $(DF)^2 - (DF)^2 (PF)^2 = (PF)^2$

$$(DF)^2 \left[1 - (PF)^2\right] = (PF)^2$$

$$(DF)^2 = \frac{(PF)^2}{\left[1 - (PF)^2\right]}$$

And finally: $DF = \frac{PF}{\sqrt{1 - (PF)^2}}$

Computations:

PF	DF	Divergence Error
0	0	0
.001	.0010000005	.00005 %
.01	.0100005	.005 %
.10	.1005	.5 %
.20	.204	2.0 %

For all values of DF=.1 (10% DF) or less, the error in the assumption DF=PF will be 0.5% or less.

The relationships between PF and DF, with AC voltage applied, and the factors that are concerned in these figures are delineated in the following:

where;

R=equivalent series resistance (ohms)

$X = (X_C - X_L)$ = total reactance (ohms)

$X_C = \frac{1}{2\pi fC}$ = capacitive reactance (ohms)

C=capacitance (farads)

f=frequency (Hz)

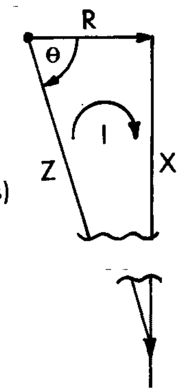
$X_L = 2\pi fL$ = inductive reactance (ohms)

L=inductance (henries)

$Z^2 = R^2 + X^2$ = impedance (ohms)

I=current (amperes)

θ=phase angle (radians or degrees)



Ohm's Law equations are:

$$E = IZ$$
$$E_X = IX$$
$$E_R = IR$$

where E_R=series resistance voltage

E_X=reactance voltage

E=circuit voltage

So: $PF = \frac{\text{Power Loss}}{\text{Total Power}} = \frac{E_R I}{EI} = \frac{I^2 R}{I^2 Z} = \frac{R}{Z} = \cos\theta$

The $\cos\theta$ and $\cot\theta$ approach convergence as θ approaches 90°. At 90°, both $\cos\theta$ and $\cot\theta = 0$ and Z = X.

From $\cos\theta = 0$ to $\cos\theta = .1$ (10%), the divergence error of the equation $\cos\theta = \cot\theta$ goes from 0 to .5% error.

∴ For values of PF = $\cos\theta = .1$ (10%) or less, we equate $\cos\theta$ and $\cot\theta$

Thus: $PF = \cos\theta \cong \cot\theta = \frac{R}{X} = DF$

In the case of a capacitor, particularly in the low frequency range (30Khz and below), the X_L term is extremely small compared to X_C and can be ignored for computation purposes. This is best illustrated by the following typical example of a metallized polyester dielectric capacitor.

Electrical Measurements: (@ 1000 Hz) Capacitance = 1.0 mf Inductance = .03 mh
$X_C = \frac{1}{2\pi fC} = \frac{1}{(6.28)(10^3)\,10^{-6})} = \frac{10^3}{6.28} = 160$ ohms
$X_L = 2\pi fL = (6.28)(10^3)\,3\times10^{-8} = .00188$ ohms
$\therefore DF = \frac{R}{(X_C - X_L)} \cong \frac{R}{X_C} = 2\pi fCR$

The DF figure will vary with frequency, capacitance, and series resistance. The DF figure will also vary with whatever environmental conditions cause C and R to change, such as temperature, moisture, pressure, etc. Figures 1 & 2 (next page) compare the DF vs. Temperature characteristics of some of the commonly used capacitor dielectrics. These plots are average curves and should not be construed as specific or absolute values since special additives, fillers and special procedures can change the curves considerably.

Mica and glass dielectric capacitors generally have DF values between .03 to 1.0% DF over the full temperature range.

Ceramic dielectric units can be very stable or extremely erratic depending on the dielectric constant (K) value of the ceramic mixture. NPO type units (low K values) will generally measure between 0.1% to 0.5% DF at room temperature, while the General Purpose type (high K values) generally read between 1.0% to 2.5% DF at room temperature. Most electrolytic capacitors have PF values that exceed 10% and therefore the relation DF=PF would not be valid. An exception to this could be the "solid" type tantalum line that will hold between 3.0 to 6.0% PF over the temperature range of -55°C to +85°C. The accurate determination of the DF vs. Frequency characteristic of capacitors, particularly at the upper frequency range, is highly influenced by testing equipment and procedures.

The reason for using DF as a parameter, instead of PF, is that determination of PF values requires much more complicated equipment and procedures than the simple comparison technique used for measuring DF.

DF then is a convenient, somewhat artificial method by which the "inefficiency" of a capacitor can be noted.

Except in a few special cases where even a few Ohms of series resistance becomes critical (extreme cases of discharge times), the value of the DF is of no real importance in the operation of an essentially DC circuit (i.e. pure DC or a small AC ripple superimposed on a polarized DC voltage).

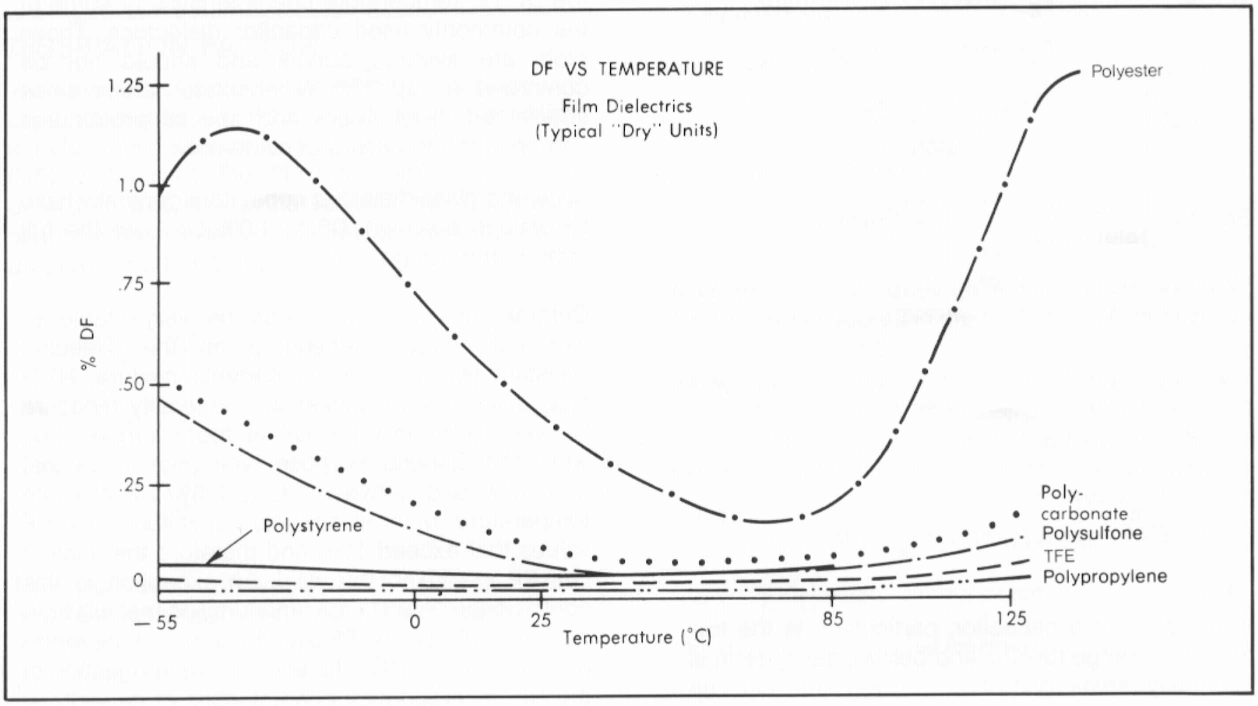


Figure 1

The manufacturer can and does use the DF measurement as a quality tool. Variations in DF above normal values for a particular line, or lot of units, would indicate possible loss of control on materials or manufacturing procedures. By monitoring the DF measurements, possible complications in the production line can be discovered quickly and corrective action instituted before the trouble reaches catastrophic proportions.

Q (FACTOR OF MERIT)

The "Q" or "Factor of Merit" of a device is also an artificial measurement that will conveniently allow notation of the "inefficiency" (or power losses) of that device.

By definition, Q is the ratio of the reactance to the series resistance. Therefore showing the following relationship.

$$Q = \frac{X}{R} = \frac{1}{DF}$$

or: Q is the reciprocal of the DF.

For instance:

$$DF = 1.0\% \quad \text{then} \quad Q = \frac{1}{.01} = 100$$

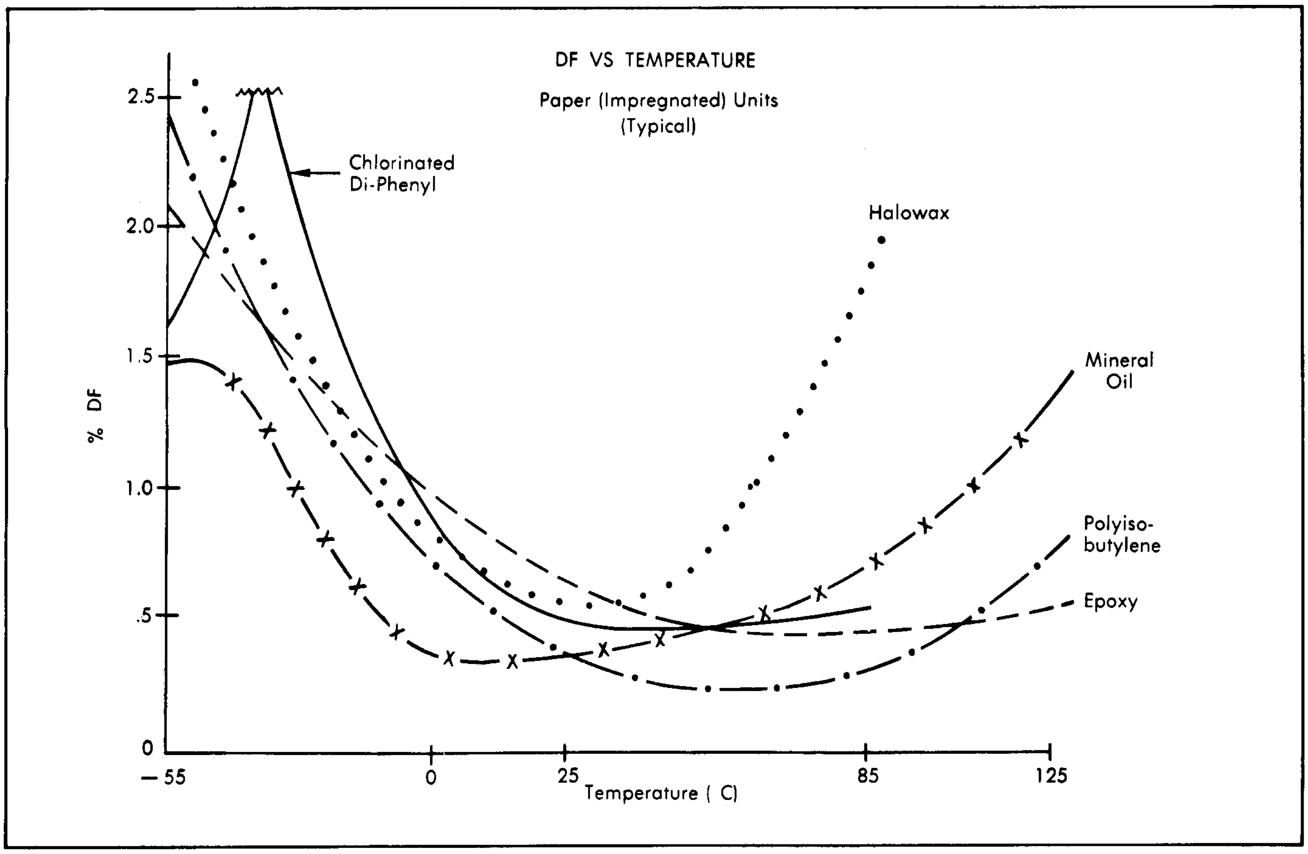


Figure 2

Conclusion:

1. Power Factor is used for capacitors when the PF is 10% or greater.
2. Dissipation Factor is used when the PF is less than 10%.
3. Q is rarely used for capacitors. (It is widely used for inductors and total circuits).